1. Coordinate transformations

1.1. 2D

Point A in Figure 1.1 has coordinates x_A and y_A in system xO_0y and X_A and Y_A in system XOY. Coordinates in one system can be calculated as functions of the coordinates in the other system according to relations (1.1) and (1.2).

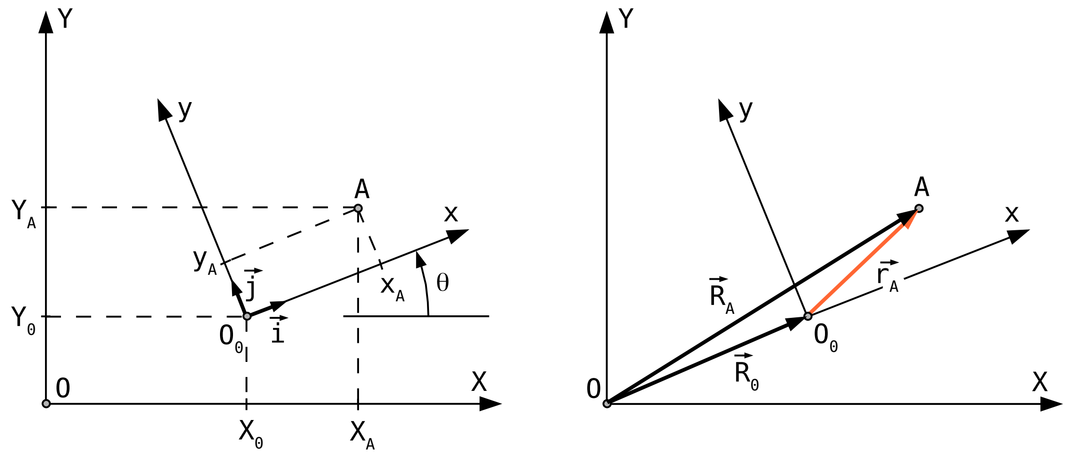

Figura 1.1 - Coordinates of a point in 2 systems of axes

$$\vec{i}=\begin{pmatrix} i_X \\ i_Y \end{pmatrix}=\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \quad ; \quad \vec{j}=\begin{pmatrix} j_X \\ j_Y \end{pmatrix}=\begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix} \quad ; \quad [C]=\begin{pmatrix} \vec{i} & \vec{j} \end{pmatrix}=\begin{pmatrix} i_X & j_X \\ i_Y & j_Y \end{pmatrix}=\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \quad ;$$

$$\begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}=\vec{R}_0 \quad ; \quad \begin{pmatrix} X_A \\ Y_A \end{pmatrix}=\vec{R}_A=\vec{R}_0+\vec{r}_A \quad ; \quad \vec{r}_A=x_A\cdot\vec{i}+y_A\cdot\vec{j} \quad ;$$

$$\begin{pmatrix} X_A \\ Y_A \end{pmatrix}=\begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}+x_A\cdot\begin{pmatrix} i_X \\ i_Y \end{pmatrix}+y_A\cdot\begin{pmatrix} j_X \\ j_Y \end{pmatrix}=\begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}+\begin{pmatrix} i_X & j_X \\ i_Y & j_Y \end{pmatrix}\cdot\begin{pmatrix} x_A \\ y_A \end{pmatrix}=\begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}+[C]\cdot\begin{pmatrix} x_A \\ y_A \end{pmatrix} \tag{1.1}$$

$$\begin{pmatrix} X_A \\ Y_A \end{pmatrix}-\begin{pmatrix} X_0 \\ Y_0 \end{pmatrix}=[C]\cdot\begin{pmatrix} x_A \\ y_A \end{pmatrix}\rightarrow\begin{pmatrix} x_A \\ y_A \end{pmatrix}=[C]^{-1}\cdot\begin{pmatrix} X_A-X_0 \\ Y_A-Y_0 \end{pmatrix}=[C]^T\cdot\begin{pmatrix} X_A-X_0 \\ Y_A-Y_0 \end{pmatrix} \tag{1.2}$$

where i_X , i_Y , j_X and j_Y are projections of unit vectors $\vec{i}$ and $\vec{j}$ of axes x and y on axes X and Y.

1.2. 3D

$$[C]=\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \end{pmatrix}=\begin{pmatrix} i_X & j_X & k_X \\ i_Y & j_Y & k_Y \\ i_Z & j_Z & k_Z \end{pmatrix} \quad ; \quad \begin{pmatrix} X_A \\ Y_A \\ Z_A \end{pmatrix}=\begin{pmatrix} X_0 \\ Y_0 \\ Z_0 \end{pmatrix}+[C]\cdot\begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} \tag{1.3}$$

$$\begin{pmatrix} x_A \\ y_A \\ z_A \end{pmatrix} = [C]^T \cdot \begin{pmatrix} X_A - X_0 \\ Y_A - Y_0 \\ Z_A - Z_0 \end{pmatrix} \quad (1.4)$$